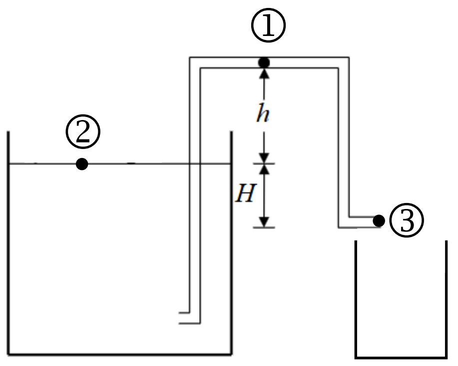

Introduction to Transport Phenomena: Mid-term Exam Solutions

<u>Question 1 – Solutions (7 points)</u>

a) Pressure at point ① (3.5 points)

To calculate the pressure at point 1, let's consider the Bernoulli equation between points ② and ①.



$$P_2 + \rho g h_2 + \frac{\rho v_2^2}{2} = P_1 + \rho g h_1 + \frac{\rho v_1^2}{2}$$

We have that:

- $v_2 = 0$ at the surface of the reservoir exposed to air
- If we take point 2 as our reference point, $h_2 = 0$ and $h_1 = h$
- $P_2 = 1\ atm$ or $P_2 = 0$ if we work in Gauge pressure

Therefore, the equation simplifies to:

$$0 = P_1 + \rho g h + \frac{\rho v_1^2}{2}$$
$$P_1 = -\rho g h - \frac{\rho v_1^2}{2}$$

We need to calculate v_1 in order to calculate P_1.

Let's consider points ① and ③.

For the continuity equation $A_1 v_1 = A_3 v_3$. Since $d_1 = d_3$ (constant pipe dimeter), $A_1 = A_3$. Thus, $v_1 = v_3$

We can calculate v_3 by applying the Bernoulli equation between points ② and ③.

$$P_2 + \rho g h_2 + \frac{\rho v_2^2}{2} = P_3 + \rho g h_3 + \frac{\rho v_3^2}{2}$$

We have that:

- $v_2 = 0$ at the surface of the reservoir exposed to air

- If we take point 2 as our reference point, $h_2 = 0$ and $h_3 = -H$
- $P_2 = P_3 = 1\ atm\ or\ P_2 = P_3 = 0$ if we work in Gauge pressure

Therefore, the equation simplifies to:

$$0 = -\rho g H + \frac{\rho v_3^2}{2}$$

$$v_3 = \sqrt{2gH} = \sqrt{2 \cdot 9.81 \left[\frac{m}{s^2}\right] \cdot 2[m]} = 6.3 \left[\frac{m}{s}\right]$$

We have then $v_1 = 6.3$ m/s and:

$$P_1 = -\rho g h - \frac{\rho v_1^2}{2} = -1000 \left[\frac{kg}{m^3}\right] \cdot 9.81 \left[\frac{m}{s^2}\right] \cdot 1[m] - \frac{1000 \left[\frac{kg}{m^3}\right] \cdot 6.3^2 \left[\frac{m^2}{s^2}\right]}{2} = -29'655[Pa]$$

The pressure is -29’655 [Pa] gauge or 71’670 [Pa] absolute at point ①.

b) How does a siphon work? (1 point)

In a siphon, there is a region (point ①) where the pressure is below atmospheric pressure. Therefore, the liquid gets suck into the piping system, thus explaining how a siphon works without any pump.

c) Maximum possible height of the siphon (1.5 points)

The pressure cannot go below absolute zero, therefore the maximum height would be achieved when $P_1 = 0$ [Pa] in absolute or -101’325 [Pa] in Gauge.

Therefore:

$$P_1 = -\rho g h - \frac{\rho v_1^2}{2}$$

$$h = \frac{-P_1 - \frac{\rho v_1^2}{2}}{\rho g} = \frac{101'325[Pa] - \frac{1000 \left[\frac{kg}{m^3}\right] \cdot 6.3^2 \left[\frac{m^2}{s^2}\right]}{2}}{1000 \left[\frac{kg}{m^3}\right] \cdot 9.81 \left[\frac{m}{s^2}\right]} = 8.31\ [m]$$

d) Maximum height of the siphon without cavitation (1 point)

The pressure limit for cavitation is the water vapor pressure which is reached when $P_1 = 3'169$ [Pa] in absolute or -98’156 [Pa] in Gauge.

$$P_1 = -\rho g h - \frac{\rho v_1^2}{2}$$

$$h = \frac{-P_1 - \frac{\rho v_1^2}{2}}{\rho g} = \frac{98'156[Pa] - \frac{1000\left[\frac{kg}{m^3}\right]\cdot 6.3^2\left[\frac{m^2}{s^2}\right]}{2}}{1000\left[\frac{kg}{m^3}\right]\cdot 9.81\left[\frac{m}{s^2}\right]} = 7.98\ [m]$$

<u>Question 2 – Solutions (11 points)</u>

General momentum balance

$$\sum F_{surface} + \sum F_{volume} = \sum_i^N \int_{A_i} \rho\bar{v}(\bar{v}\cdot\hat{n})dA_i$$

$$where \sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{volume} = \sum F_{gravity} = mg$$

The pressure forces are given by:

$$\sum_i^N F_{pressure} = -P_i . A_i . \widehat{n_i}$$

The friction force is associated to the friction losses in the flange faucet according to:

$$F_{friction} = A\cdot\Delta P_f = \frac{\pi D^2}{4}\cdot\Delta P_f$$

Where:

$$\Delta P_f = \frac{\rho v^2}{2}\left(\frac{4f_f}{D}\sum L_L + \sum K_L\right)$$

We have:

1. $\sum L_L = 0.5\ m$
2. $\sum K_L = 17\ (Gate\ Valve\ Spigot) + 0.2\ (45°\ Elbow) = 17.2$
3. $D = 0.02\ m$
4. $\rho = 1000\ \frac{kg}{m^3}$

We need to calculate the velocity which is given by $v = \frac{Q}{A}$

1. $Q = 1.17\left[\frac{L}{s}\right] = \frac{1.17\left[\frac{L}{s}\right]}{1000\left[\frac{L}{m^3}\right]} = 1.17\cdot 10^{-3}\left[\frac{m^3}{s}\right]$
2. $A = \frac{\pi D^2}{4} = \frac{3.14\cdot(0.02\text{m})^2}{4} = 3.14\cdot 10^{-4}[m^2]$

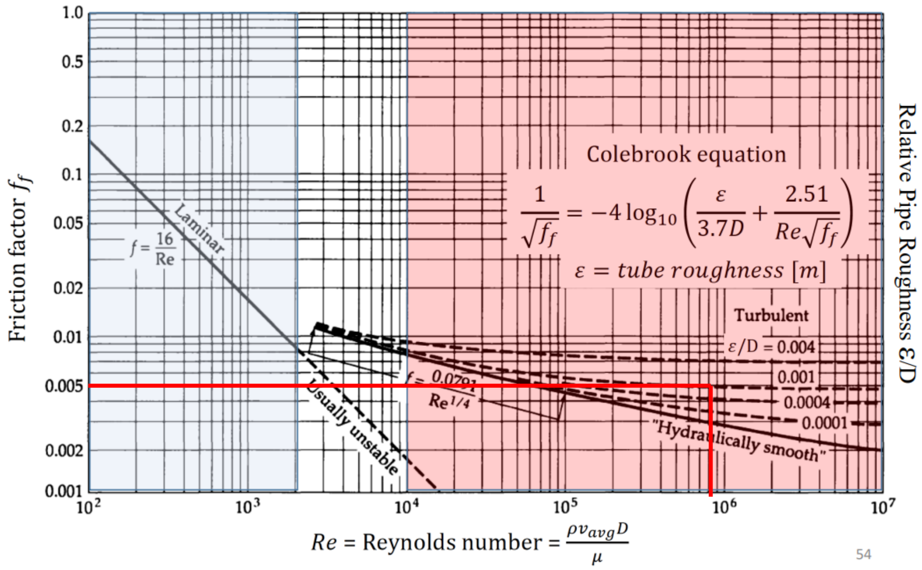
$$v = \frac{1.17 \cdot 10^{-3} \left[\frac{m^3}{s}\right]}{3.14 \cdot 10^{-4} \, [\,m^2]} = 3.73 \left[\frac{m}{s}\right]$$

The final thing to do is to calculate the friction factor f_f with the Moody diagram:

$$Re = \frac{\rho v D}{\mu} = \frac{1000 \left[\frac{kg}{m^3}\right] \cdot 3.73 \left[\frac{m}{s}\right] \cdot 0.02 \, [m]}{9 \cdot 10^{-4} \left[\frac{kg}{m \cdot s}\right]} = 82'888$$

The flow is turbulent. The relative pipe roughness is $\frac{\varepsilon}{D} = \frac{2 \cdot 10^{-5} \text{ m}}{0.02 \, m} = 10^{-3}$

We can read from the Moody diagram, $f_f \approx 0.005$



We finally have:

$$\Delta P_f = \frac{1000 \frac{kg}{m^3} \left(3.73 \frac{m}{s}\right)^2}{2} \left(\frac{4 \cdot 0.005}{0.02 \, m} \cdot 0.5m + 17.2\right) = 123'130 \, [Pa]$$

$$F_{friction} = \frac{\pi D^2}{4} \cdot \Delta P_f = 3.14 \cdot 10^{-4} [m^2] \cdot 123'130 \, [Pa] = 38.7 \, [N]$$

Based on the instructions, 19.3 [N] are applied horizontally, and 19.3 [N] are applied vertically.

Remember that the friction force is acting against the fluid flow, so in the -x and +y direction.

Solving the general momentum balance in the x-directions gives:

$$F_{friction,x} + F_{pressure,x} + F_{reaction,x} = \sum_i^N \int_{A_i} \rho \bar{v} (\bar{v} \cdot \hat{n}) dA_i$$

$$F_{reaction,x} = \sum_{i}^{N} \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n}) dA_i - F_{friction,x} - F_{pressure,x}$$

$$F_{reaction,x} = \rho \bar{v} A(\bar{v} \cdot \widehat{n_1}) - F_{friction,x} - (-P_i \cdot \mathrm{A} \cdot \widehat{n_\imath})$$

$$F_{reaction,x} = \rho v^2 \mathrm{A} \cdot (-1) - (-19.3\, N) - (-P_i \cdot \mathrm{A} \cdot (-1))$$

$$F_{reaction,x} = -1000 \frac{kg}{m^3} \cdot \left(3.73 \frac{m}{s}\right)^2 \cdot 3.14 \cdot 10^{-4}\, m^2 + 19.3\, N - 90'000\, \mathrm{Pa} \cdot 3.14 \cdot 10^{-4}\, m^2$$

$$F_{reaction,x} = -13.3\, [N]$$

Solving the general momentum balance in the y-directions gives:

$$F_{friction,y} + F_{pressure,y} + F_{reaction,y} + F_{gravity} = \sum_{i}^{N} \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n}) dA_i$$

The pressure at the exit of the faucet is the atmospheric pressure. Since the atmospheric pressure is 0 in gauge, we do not have any pressure term in the y-direction.

$$F_{reaction,y} = \sum_{i}^{N} \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n}) dA_i - F_{friction,y} - F_{gravity}$$

$$F_{reaction,y} = \rho \bar{v} A(\bar{v} \cdot \widehat{n_2}) - F_{friction,y} - m\vec{g}$$

$$F_{reaction,y} = -\rho v^2 \mathrm{A} \cdot -19.3\, N + mg$$

$$F_{reaction,y} = -1000 \frac{kg}{m^3} \cdot \left(3.73 \frac{m}{s}\right)^2 \cdot 3.14 \cdot 10^{-4}\, m^2 - 19.3\, N + 5.7\, kg \cdot 9.81\, \frac{m}{s^2}$$

$$F_{reaction,y} = 32.2\, [N]$$

<u>Question 3 – Solution (10 points)</u>

a) Calculation of $\dot{Q}$ and $T_{h,out}$ (2 points)

From the heat balance:

$$\dot{Q}_h = \dot{Q}_c = \left[\dot{m} c_p (T_{in} - T_{out})\right]_h = \left[\dot{m} c_p (T_{out} - T_{in})\right]_c$$

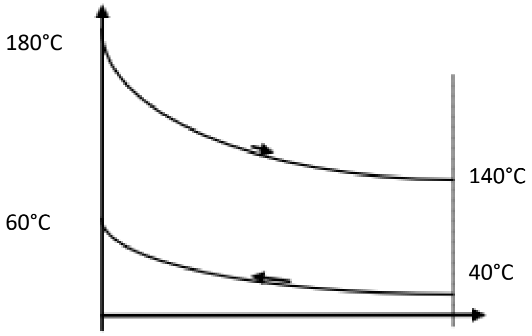
We have:

$$\dot{Q}_c = \left[\dot{m} c_p (T_{out} - T_{in})\right]_c = \frac{20'000 \left[\frac{kg}{h}\right]}{3600 \left[\frac{s}{h}\right]} \cdot 4181 \left[\frac{J}{kg \cdot K}\right] \cdot (60 - 40)\, [K] = 464'556\, [W]$$

So:

$$T_{h,out} = T_{h,in} - \frac{\dot{Q}_c}{\dot{m}c_p} = 180\ [°C] - \frac{464'556\ [W]}{\frac{10'000\left[\frac{kg}{h}\right]}{3600\ \left[\frac{s}{h}\right]} \cdot 4181 \left[\frac{J}{kg \cdot K}\right]} = 140.00\ [°C]$$

b) Temperature profile (1 points)



1) Calculation of the heat transfer area $A_{exchange}$ (3 points)

$$\Delta T_1 = T_{h,in} - T_{c,out} = 180°C - 60°C = 120°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 140°C - 40°C = 100°C$$

$$\Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(120°C - 100°C)}{ln\left(\frac{120°C}{100°C}\right)} = 109.70\ [K]$$

$$A_{ex} = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{464'556\ [\mathrm{W}]}{450\ \left[\frac{W}{m^2 \cdot K}\right] \cdot 109.70\ [K]} = 9.411\ [m^2]$$

2) Calculation of v (1.5 points)

By the definition of Reynolds number:

$$Re = \frac{\rho v D}{\mu} = 10'000$$

So :

$$v = \frac{Re \cdot \mu}{\rho D} = \frac{10'000 \cdot 1.704 \cdot 10^{-4} [Pa \cdot s]}{908.3\ \left[\frac{kg}{m^3}\right] \cdot 0.02\ [m]} = 0.0938\ \left[\frac{m}{s}\right]$$

3) Number of tubes and heat-exchanger length (2.5 points)

The cross-sectional area of a single tube is given by:

$$A_{c,tube} = \frac{\pi D^2}{4} = \frac{3.14 \cdot (0.02\mathrm{m})^2}{4} = 3.14 \cdot 10^{-4} [\ m^2]$$

Therefore, the total number of tubes is given by:

$$N_{tube} = \frac{A_c}{A_{c,tube}} = \frac{0.033\ [m^2]}{3.14 \cdot 10^{-4} [\ m^2]} = 105.1 \cong 106$$

106 tubes are required in the heat exchanger.

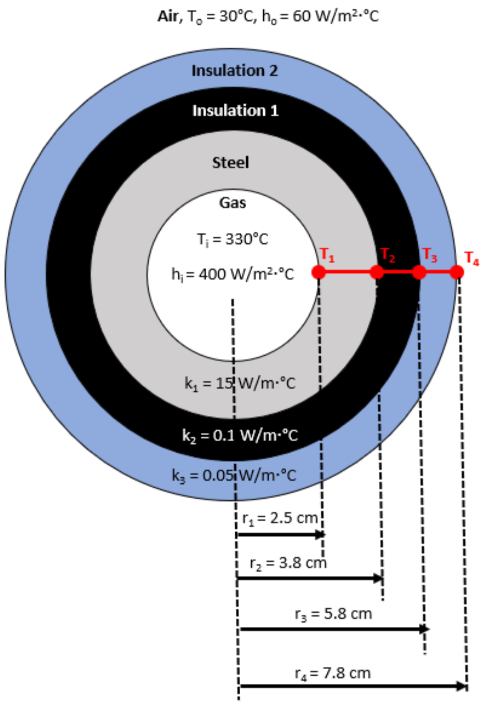
The length of the heat exchanger is obtained by using the total surface area of the tubes:

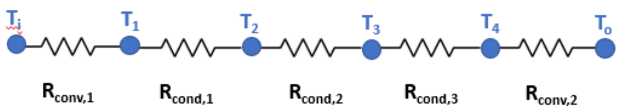
$$A_{ex} = 106 \cdot A_{ex,tube} = 106 \cdot \pi \cdot D \cdot L$$

$$L = \frac{A_{ex}}{106 \cdot \pi \cdot D} = \frac{9.411\ [m^2]}{106 \cdot 3.14 \cdot 0.02\ [m]} = 1.41\ [m]$$

<u>Question 4 – Solution (12 points)</u>

a) Pipe scheme and thermal resistance circuit (1.5 points)





b) Calculation of the heat loss (5.5 points)

The heat loss is given by:

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{T_i - T_o}{R_{total}}$$

Where :

$$R_{total} = R_{conv,1} + R_{cond,1} + R_{cond,2} + R_{cond,3} + R_{conv,2}$$

We have :

$$R_{conv,1} = \frac{1}{2\pi r_1 L h_i} = \frac{1}{2\pi \cdot 0.025[m] \cdot 10[m] \cdot 400\left[\frac{W}{m^2K}\right]} = 1.59 \cdot 10^{-3}\left[\frac{K}{W}\right]$$

$$R_{cond,1} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_1} = \frac{ln\left(\frac{3.8\ [cm]}{2.5\ [cm]}\right)}{2\pi \cdot 10[m] \cdot 15\left[\frac{W}{mK}\right]} = 4.44 \cdot 10^{-4}\left[\frac{K}{W}\right]$$

$$R_{cond,2} = \frac{ln\left(\frac{r_3}{r_2}\right)}{2\pi L k_2} = \frac{ln\left(\frac{5.8\ [cm]}{3.8\ [cm]}\right)}{2\pi \cdot 10[m] \cdot 0.1\left[\frac{W}{mK}\right]} = 6.73 \cdot 10^{-2}\left[\frac{K}{W}\right]$$

$$R_{cond,3} = \frac{ln\left(\frac{r_4}{r_3}\right)}{2\pi L k_3} = \frac{ln\left(\frac{7.8\ [cm]}{5.8\ [cm]}\right)}{2\pi \cdot 10[m] \cdot 0.05\left[\frac{W}{mK}\right]} = 9.43 \cdot 10^{-2}\left[\frac{K}{W}\right]$$

$$R_{conv,2} = \frac{1}{2\pi r_4 L h_o} = \frac{1}{2\pi \cdot 0.078[m] \cdot 10[m] \cdot 60\left[\frac{W}{m^2K}\right]} = 3.40 \cdot 10^{-3}\left[\frac{K}{W}\right]$$

So :

$$R_{total} = 1.59 \cdot 10^{-3} + 4.44 \cdot 10^{-4} + 6.73 \cdot 10^{-2} + 9.43 \cdot 10^{-2} + 3.40 \cdot 10^{-3} = 1.67 \cdot 10^{-1}\left[\frac{K}{W}\right]$$

Finally:

$$\dot{Q} = \frac{T_i - T_o}{R_{total}} = \frac{330°C - 30°C}{1.67 \cdot 10^{-1}\left[\frac{K}{W}\right]} = 1796\ [W]$$

c) Calculation of the interfacial temperatures (3 points)

We have the following relationships:

$$\dot{Q} = \frac{T_i - T_1}{R_{conv,1}} = \frac{T_1 - T_2}{R_{cond,1}} = \frac{T_2 - T_3}{R_{cond,2}} = \frac{T_3 - T_4}{R_{cond,3}}$$

So:

$$T_1 = T_i - \dot{Q}R_{conv,1} = 330°C - 1796\ [W] \cdot 1.59 \cdot 10^{-3}\left[\frac{K}{W}\right] = 327.1°C$$

$$T_2 = T_1 - \dot{Q}R_{cond,1} = 327.1°C - 1796\ [W] \cdot 4.44 \cdot 10^{-4}\left[\frac{K}{W}\right] = 326.3°C$$

$$T_3 = T_2 - \dot{Q}R_{cond,2} = 326.3°C - 1796\ [W] \cdot 6.73 \cdot 10^{-2} \left[\frac{K}{W}\right] = 205.4°C$$

$$T_4 = T_3 - \dot{Q}R_{cond,3} = 205.4°C - 1796\ [W] \cdot 9.43 \cdot 10^{-2} \left[\frac{K}{W}\right] = 36.0°C$$

We can check our result by using the outside convection resistance.

$$\dot{Q} = \frac{T_4 - T_o}{R_{conv,2}}$$

$$T_4 = T_i + \dot{Q}R_{conv,2} = 30°C + 1796[W] \cdot 3.4 \cdot 10^{-3} \left[\frac{K}{W}\right] = 36.1°C$$

Everything is consistent.

d) Is the second insulating layer useful ? (2 points)

If we remove the second insultation layer, we would have:

$$R_{total} = 1.59 \cdot 10^{-3} + 4.44 \cdot 10^{-4} + 6.73 \cdot 10^{-2} + 3.40 \cdot 10^{-3} = 7.27 \cdot 10^{-2} \left[\frac{K}{W}\right]$$

$$\dot{Q} = \frac{T_i - T_o}{R_{total}} = \frac{330°C - 30°C}{7.27 \cdot 10^{-2} \left[\frac{K}{W}\right]} = 4126\ [W]$$

$$T_4 = T_i + \dot{Q}R_{conv,2} = 30°C + 4126\ [W] \cdot 3.40 \cdot 10^{-3} \left[\frac{K}{W}\right] = 44.0°C$$

The presence of the second insultation layer is useful as it allows to drastically decrease the heat loss. In the presence of second insulation layer, the heat loss decreases by factor of 2.3, specifically from 4126 [W] to 1796 [W]. In addition, the outside interface temperature decreases from 44.0°C to 36.0°C.